17. Oscillations

17.1 Simple harmonic oscillations

Paper 4

Question Paper

1 **(b)** A small ball is held in place using a stretched string. One end of the string is fixed to a wall and the other end is attached to a vibration generator, as shown in Fig. 4.1.

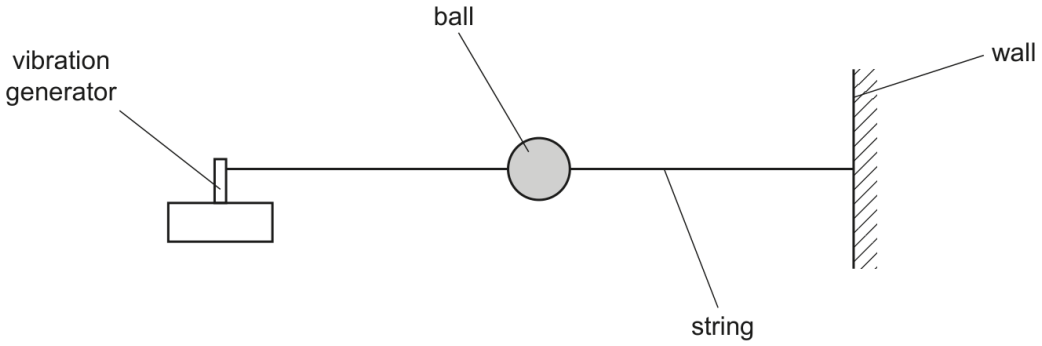


Fig. 4.1

Initially, the vibration generator is switched off.

A student displaces the ball vertically and then releases it. Fig. 4.2 shows the variation of the displacement of the ball with time after it is released.

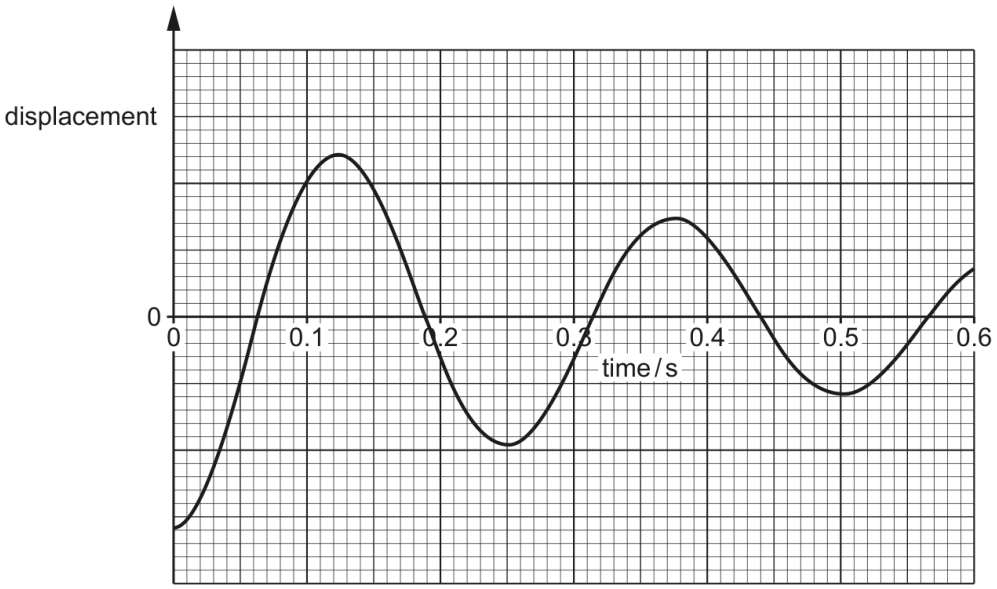


Fig. 4.2

(iii) Determine the frequency of the oscillations of the ball.

frequency = ... Hz [1]

2 A block of mass *m* oscillates vertically on a spring, as shown in Fig. 4.1.

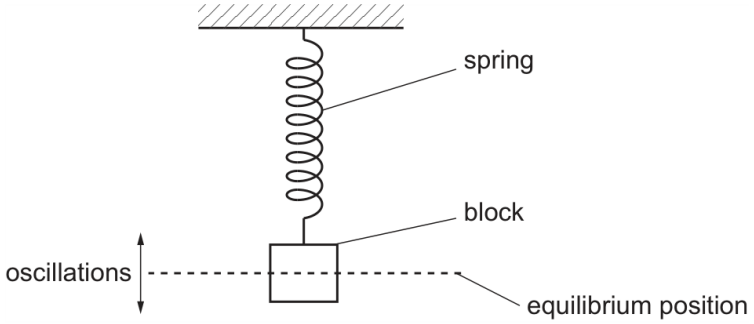


Fig. 4.1

The acceleration *a* of the block varies with displacement *x* from its equilibrium position, as shown in Fig. 4.2.

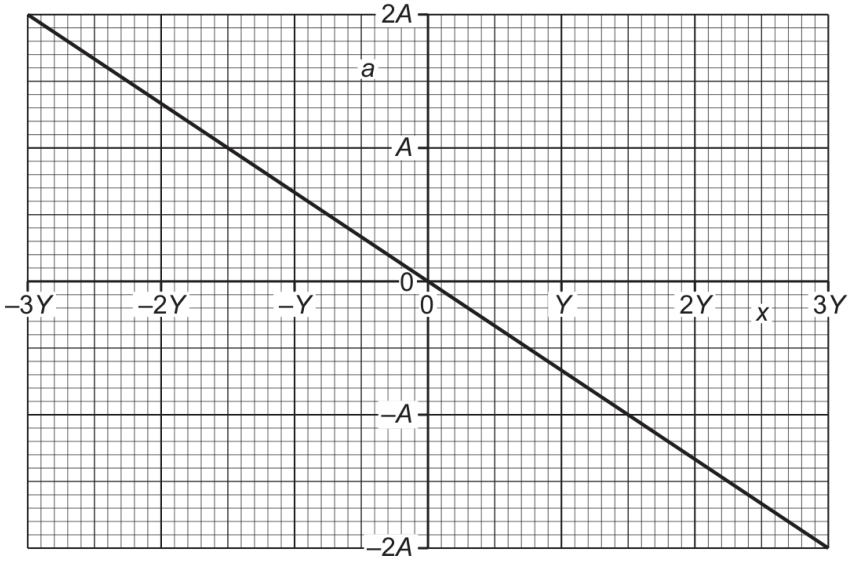


Fig. 4.2

The amplitude of the oscillations is 3*Y* and the maximum acceleration is 2*A*.

(a) Explain how Fig. 4.2 shows that the oscillations of the block are simple harmonic.

.....

.....

..... [2]

(b) Deduce expressions, in terms of some or all of m, A and Y, for:

(i) the angular frequency ω of the oscillations

ω = ... [1]

(ii) the maximum speed v_0 of the oscillations

v_0 = ... [2]

(c) The period of the oscillations is 0.75 s and the value of $3Y$ is 1.8 cm.

Determine an expression for x in terms of time t, where x is in cm and t is in seconds.

x = ... [2]

3 **(a)** State what is meant by simple harmonic motion.

.....

.....

..... [2]

(b) A block is suspended from a spring, as shown in Fig. 4.1.

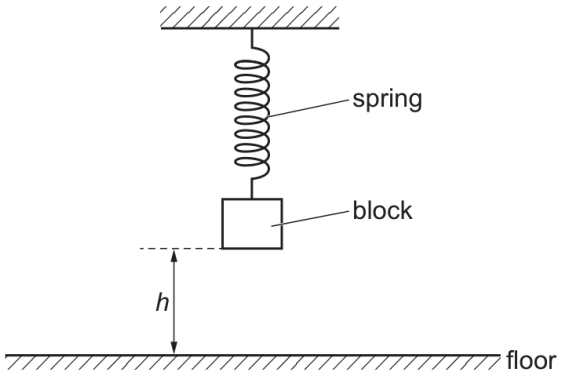


Fig. 4.1

The block is pulled down and released at time $t = 0$. It then oscillates vertically with simple harmonic motion.

Fig. 4.2 shows the variation of the velocity v of the block with height h of the base of the block above the floor.

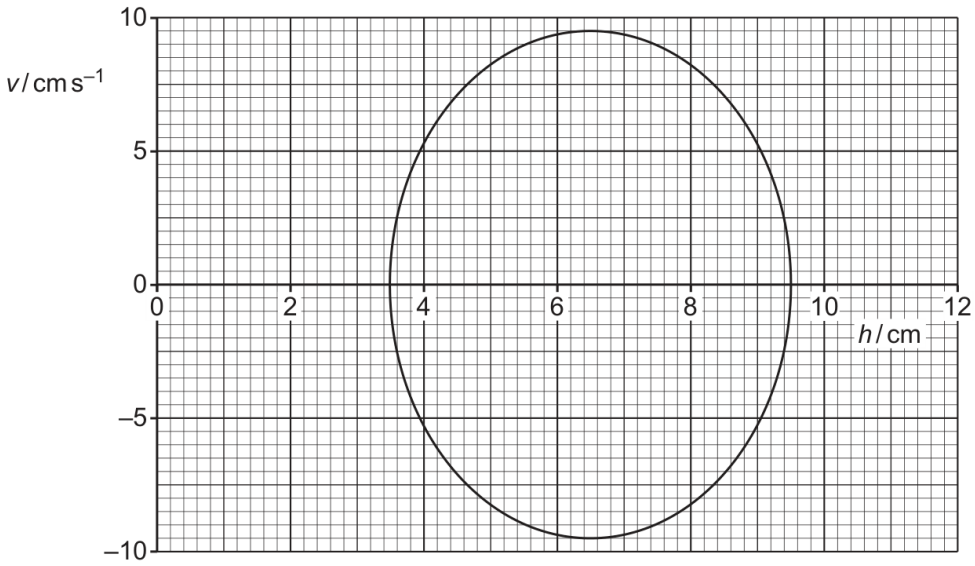


Fig. 4.2

(i) Determine the amplitude, in cm, of the oscillations.

amplitude = ... cm [1]

(ii) Show that the angular frequency of the oscillations is 3.2 rad s^{-1}.

[2]

(iii) Calculate the period T of the oscillations.

T = ... s [2]

(iv) On Fig. 4.3, sketch the variation of h with time t from $t = 0$ to $t = 6.0$ s.

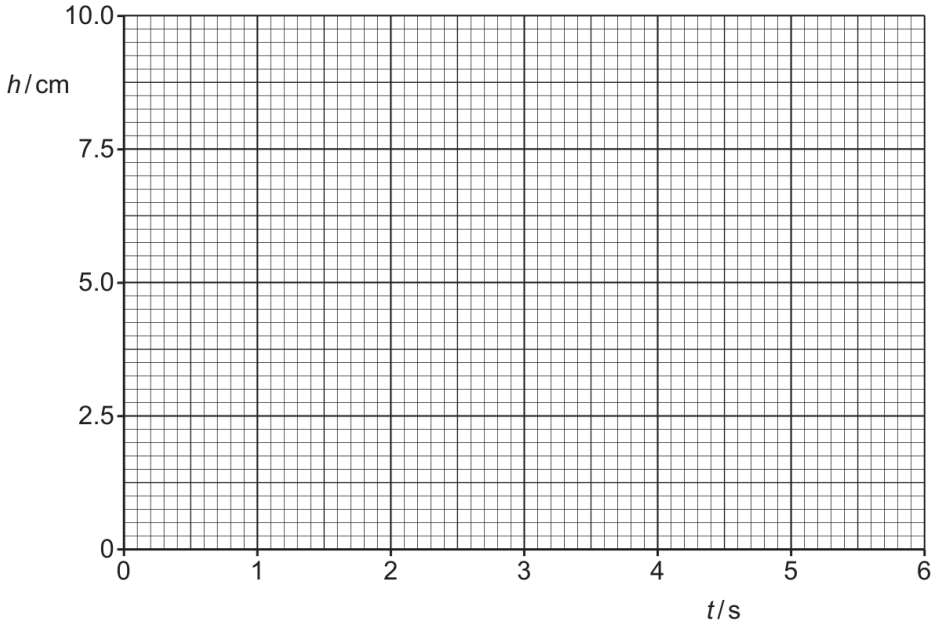


Fig. 4.3

[4]

4 Fig. 5.1 shows a pendulum consisting of a metal sphere suspended by a thin string.

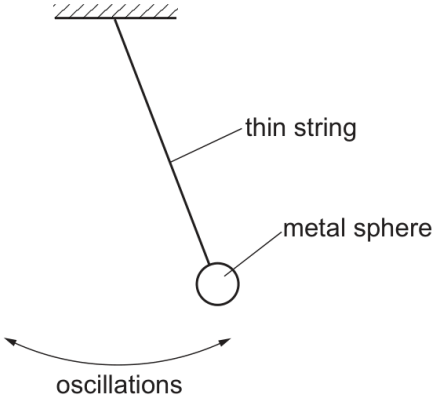


Fig. 5.1 (not to scale)

The sphere undergoes small oscillations about its equilibrium position. The oscillations may be considered to be simple harmonic.

Fig. 5.2 shows the variation with time t of the displacement x of the sphere from its equilibrium position.

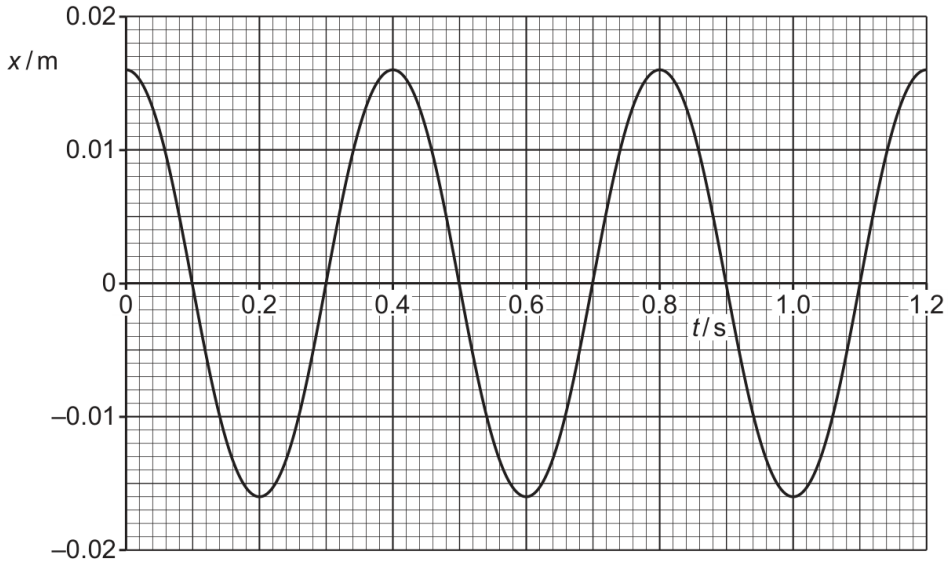


Fig. 5.2

(a) On Fig. 5.1, draw an arrow, from the centre of the sphere, to represent the direction of the resultant force acting on the sphere when it is in the position shown. [1]

(b) The mass of the sphere is 0.15 kg.

(i) State the amplitude of the oscillations.

amplitude = ... m [1]

(ii) Determine the angular frequency of the oscillations.

angular frequency = ... rad s^{-1} [2]

5 A small steel sphere is oscillating vertically on the end of a spring, as shown in Fig. 4.1.

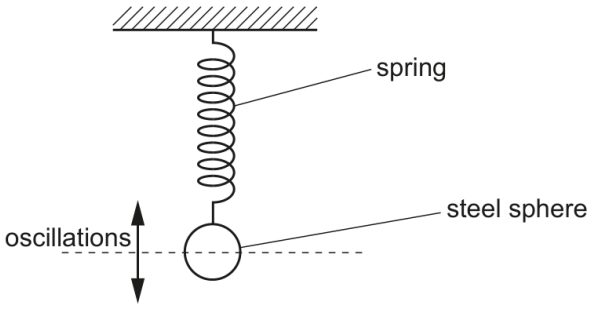


Fig. 4.1

The velocity v of the sphere varies with displacement x from its equilibrium position according to

$$v = \pm 9.7\sqrt{(11.6 - x^2)}$$

where v is in cm s^{-1} and x is in cm.

(a) (i) Calculate the frequency of the oscillations.

frequency = ... Hz [2]

(ii) Show that the amplitude of the oscillations is 3.4 cm.

[1]

(iii) Calculate the maximum acceleration a_0 of the sphere.

a_0 = ... m s^{-2} [2]

(b) On Fig. 4.2, sketch the variation with x of the acceleration a of the sphere.

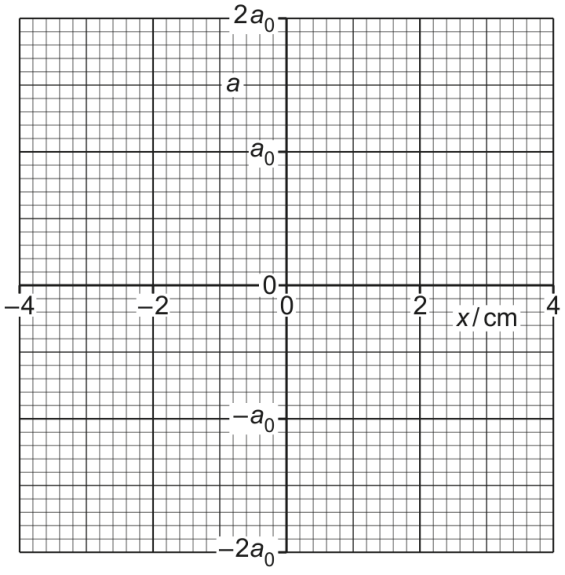


Fig. 4.2

[3]

6 A heavy metal sphere of mass 0.81 kg is suspended from a string. The sphere is undergoing small oscillations from side to side, as shown in Fig. 4.1.

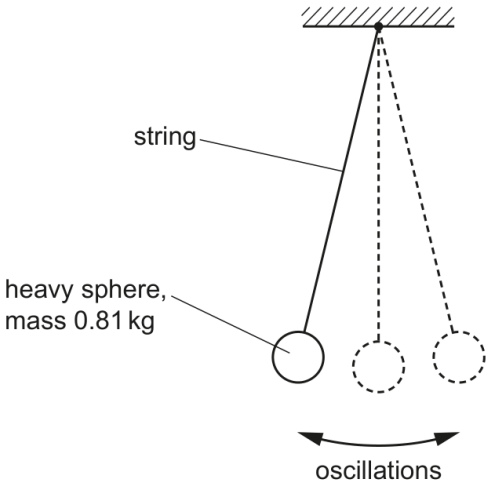


Fig. 4.1

The oscillations of the sphere may be considered to be simple harmonic with amplitude 0.036 m and period 3.0 s.

(a) State what is meant by simple harmonic motion.

.....

.....

..... [2]

(b) Calculate:

(i) the angular frequency of the oscillations

angular frequency = ... rad s^{-1} [2]

7 An electron in a metal rod moves randomly about a mean position. When an alternating voltage is applied to the ends of the rod, the mean position can be considered to oscillate with simple harmonic motion along the axis of the rod. Fig. 4.1 shows the variation with time t of the displacement x of the mean position from a fixed point on the axis of the rod.

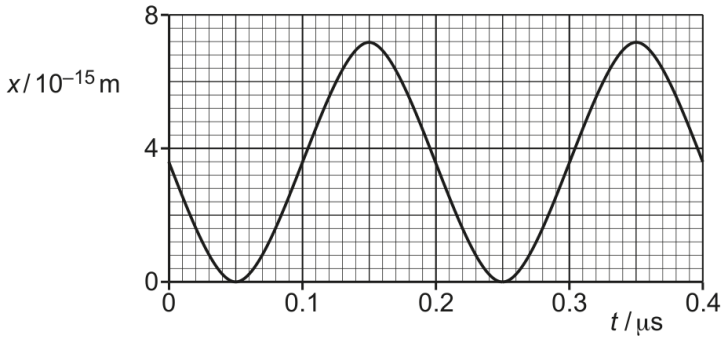


Fig. 4.1

(a) (i) Determine the amplitude of the oscillations.

amplitude = ... m [1]

(ii) Determine the angular frequency of the oscillations.

angular frequency = ... rad s^{-1} [1]

(iii) Use your answers in **(a)(i)** and **(a)(ii)** to show that the maximum drift speed v_0 of the electron is 1.1×10^{-7} m s^{-1}.

[2]

8 A pendulum consists of a bob (small metal sphere) attached to the end of a piece of string. The other end of the string is attached to a fixed point. The bob oscillates with small oscillations about its equilibrium position, as shown in Fig. 4.1.

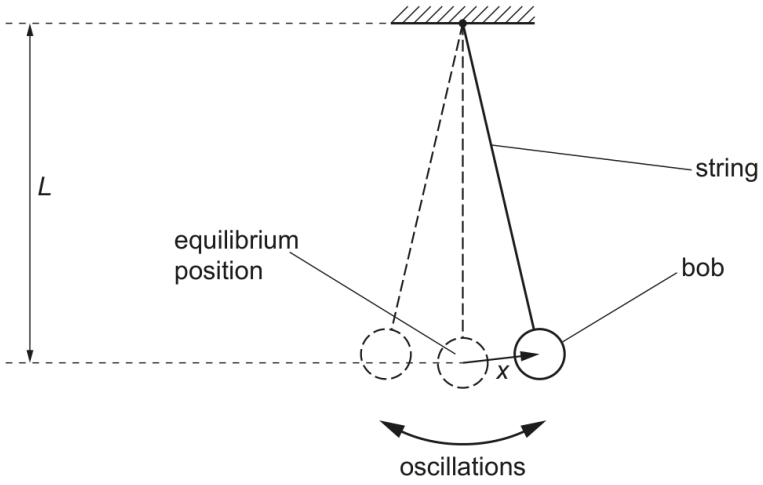


Fig. 4.1 (not to scale)

The length L of the pendulum, measured from the fixed point to the centre of the bob, is 1.24 m.

The acceleration a of the bob varies with its displacement x from the equilibrium position as shown in Fig. 4.2.

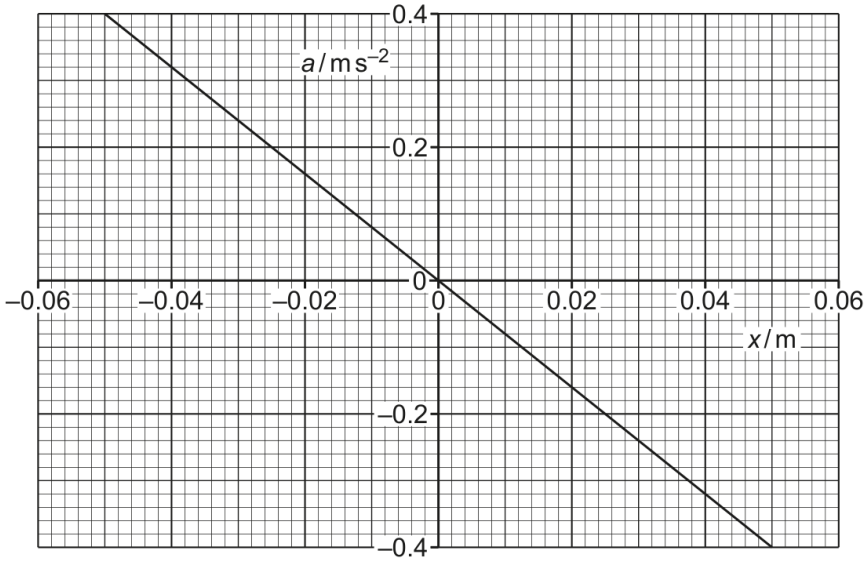


Fig. 4.2

(a) State how Fig. 4.2 shows that the motion of the pendulum is simple harmonic.

.....

.....

..... [2]

(b) (i) Use Fig. 4.2 to determine the angular frequency ω of the oscillations.

ω = ... rad s^{-1} [2]

(ii) The angular frequency ω is related to the length L of the pendulum by

$$\omega = \sqrt{\frac{k}{L}}$$

where k is a constant.

Use your answer in **(b)(i)** to determine k. Give a unit with your answer.

k = ... unit ..... [2]

9 (b) Fig. 4.1 shows a heavy pendulum and a light pendulum, both suspended from the same piece of string. This string is secured at each end to fixed points.

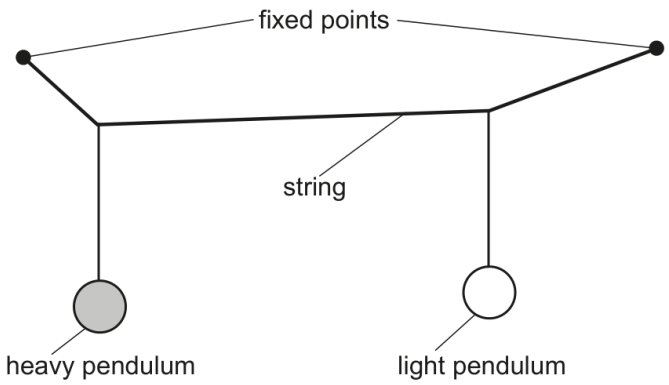


Fig. 4.1

Both pendulums have the same natural frequency.
The heavy pendulum is set oscillating perpendicular to the plane of the diagram. As it oscillates, it causes the light pendulum to oscillate.

Fig. 4.2 shows the variation with time t of the displacements of the two pendulums for three oscillations.

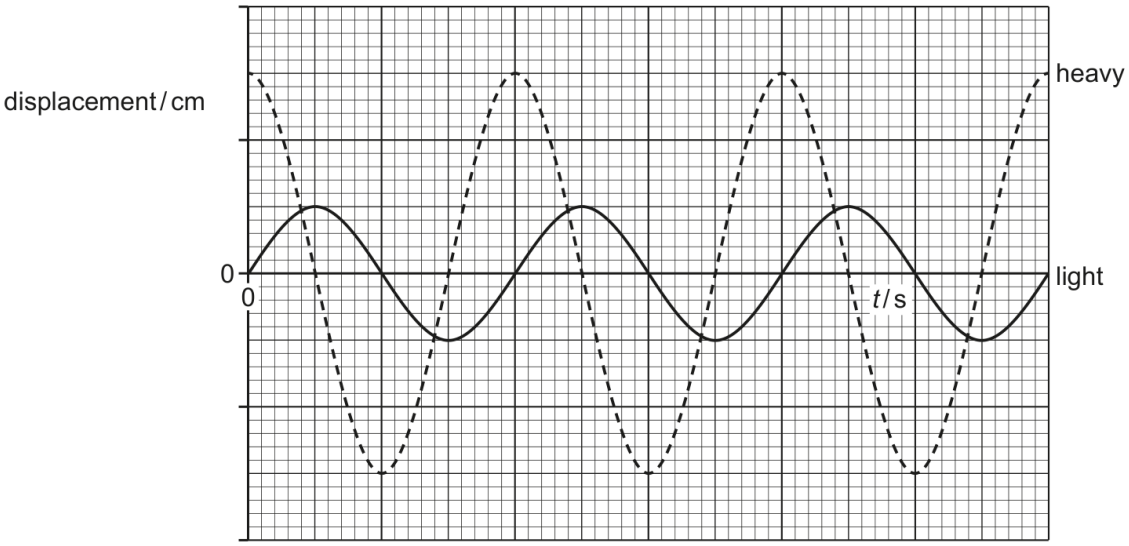


Fig. 4.2

The variation with t of the displacement x of the light pendulum is given by

$$x = 0.25 \sin 5.0\pi t$$

where x is in centimetres and t is in seconds.

(i) Calculate the period T of the oscillations.

T = ... s [2]

(ii) On Fig. 4.2, label both of the axes with the correct scales. Use the space below for any additional working that you need.

[2]

(iii) Determine the magnitude of the phase difference ϕ between the oscillations of the light and heavy pendulums. Give a unit with your answer.

ϕ = ... unit ..... [2]

10 An object is suspended from a spring that is attached to a fixed point as shown in Fig. 3.1.

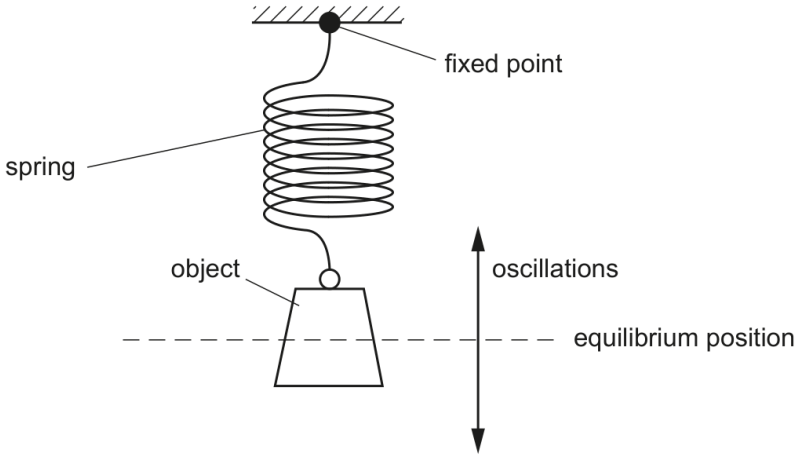


Fig. 3.1

The object oscillates vertically with simple harmonic motion about its equilibrium position.

(a) State the defining equation for simple harmonic motion. Identify the meaning of each of the symbols used to represent physical quantities.

.....

.....

..... [2]

(b) The variation with displacement x from the equilibrium position of the velocity v of the object is shown in Fig. 3.2.

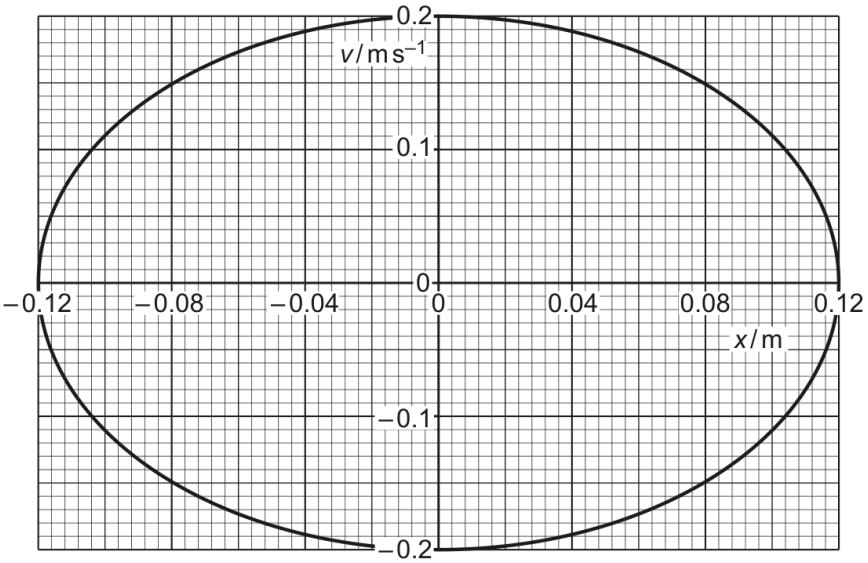


Fig. 3.2

The variation with x of the potential energy E_P of the oscillations of the object is shown in Fig. 3.3.

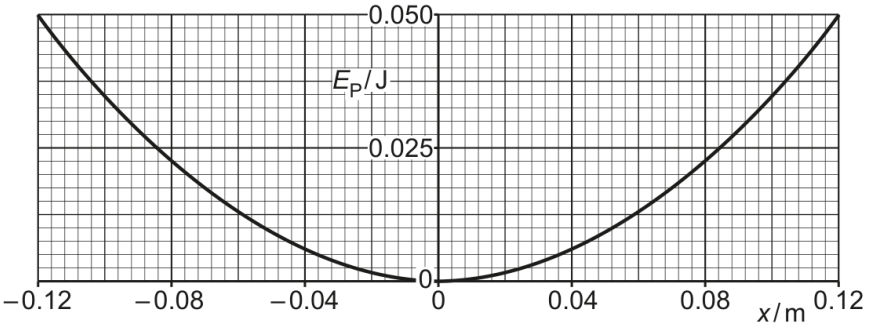


Fig. 3.3

Use Fig. 3.2 and Fig. 3.3 to:

(i) determine the amplitude x_0 of the oscillations

x_0 = ... m [1]

(ii) show that the angular frequency of the oscillations is $1.7\,\text{rad}\,\text{s}^{-1}$

[2]

(c) The oscillations of the object are now lightly damped.

(ii) Assume that the damping does not change the angular frequency of the oscillations.

On Fig. 3.2, sketch the variation with x of v when the amplitude of the oscillations is 0.060 m. [2]

11 Fig. 4.1 shows the variation with time t of the height h above the ground of an object of mass 36 kg that is undergoing vertical simple harmonic motion.

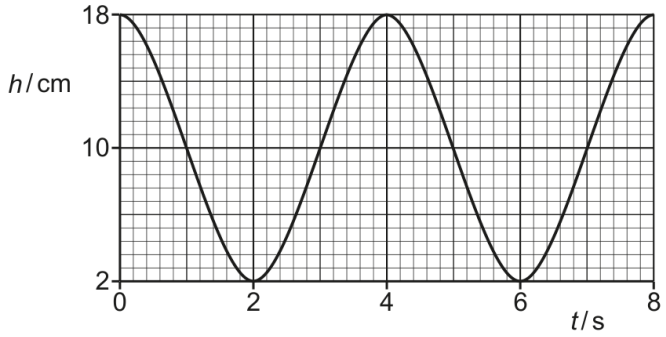


Fig. 4.1

(a) For the oscillations of the object:

(i) determine the amplitude x_0, in cm

x_0 = ... cm [1]

(ii) show that the angular frequency ω is 1.6 rad s^{-1}

[2]

12 (a) State what is meant by *simple harmonic motion*.

.....

.....

..... [2]

(b) A trolley of mass *m* is held on a horizontal surface by means of two springs. One spring is attached to a fixed point P. The other spring is connected to an oscillator, as shown in Fig. 3.1.

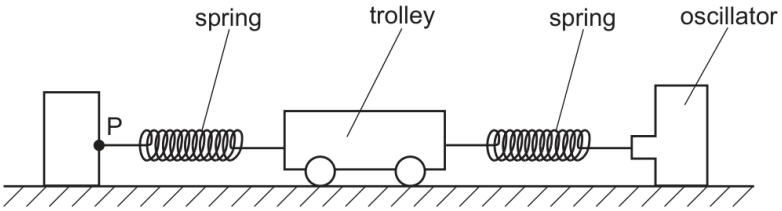


Fig. 3.1

The springs, each having spring constant *k* of 130 N m^{-1}, are always extended.

The oscillator is switched off. The trolley is displaced along the line of the springs and then released. The resulting oscillations of the trolley are simple harmonic.

The acceleration *a* of the trolley is given by the expression

$$a = -\left(\frac{2k}{m}\right)x$$

where *x* is the displacement of the trolley from its equilibrium position.

The mass of the trolley is 840 g.

Calculate the frequency *f* of oscillation of the trolley.

f = ... Hz [3]

13 A U-shaped tube contains some liquid. The liquid column in each half of the tube has length L, as shown in Fig. 3.1.

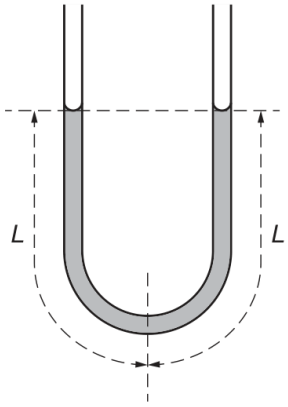


Fig. 3.1

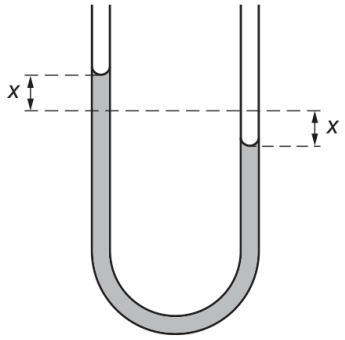


Fig. 3.2

The liquid columns are displaced vertically. The liquid then oscillates in the tube. The liquid levels are displaced from the equilibrium positions as shown in Fig. 3.2.

The acceleration a of the liquid in the tube is related to the displacement x by the expression

$$a = -\left(\frac{g}{L}\right)x$$

where g is the acceleration of free fall.

(a) Explain how the expression shows that the liquid in the tube is undergoing simple harmonic motion.

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.....

.....

.....

..... [3]

(b) The length L of each liquid column is 18 cm.

Determine the period T of the oscillations.

T = ... s [3]

14 A trolley on a track is attached by springs to fixed blocks X and Y, as shown in Fig. 4.1. The track contains many small holes through which air is blown vertically upwards. This results in the trolley resting on a cushion of air rather than being in direct contact with the track.

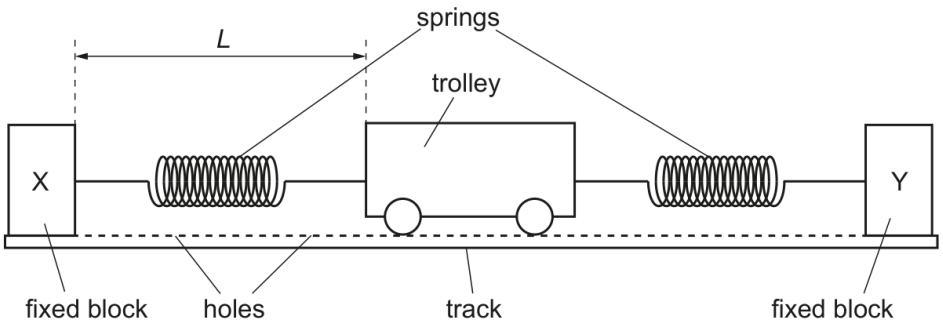


Fig. 4.1

The trolley is pulled to one side of its equilibrium position and then released so that it oscillates initially with simple harmonic motion. After a short time, the air blower is switched off. The variation with time *t* of the distance *L* of the trolley from block X is shown in Fig. 4.2.

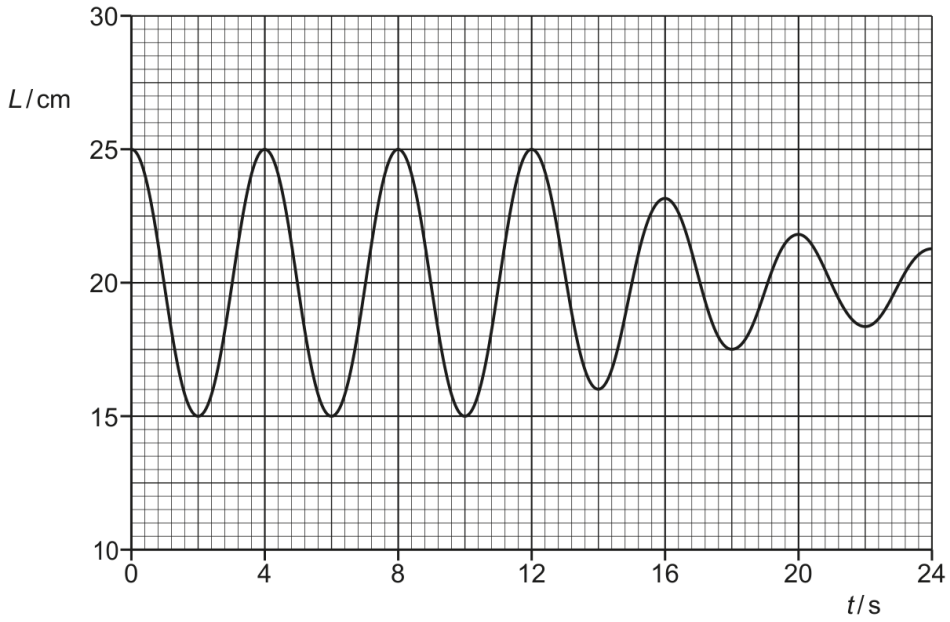


Fig. 4.2

(a) Use Fig. 4.2 to determine:

(i) the initial amplitude of the oscillations

amplitude = ... cm [1]

(ii) the angular frequency ω of the oscillations

ω = ... rad s^{-1} [2]

(iii) the maximum speed v_0, in cm s^{-1}, of the oscillating trolley.

v_0 = ... cm s^{-1} [2]

(b) Apart from the quantities in **(a)**, describe what may be deduced from Fig. 4.2 about the motion of the trolley between time t = 0 and time t = 24 s. No calculations are required.

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..... [3]

(c) On Fig. 4.3, sketch the variation with L of the velocity v of the trolley for its first complete oscillation.

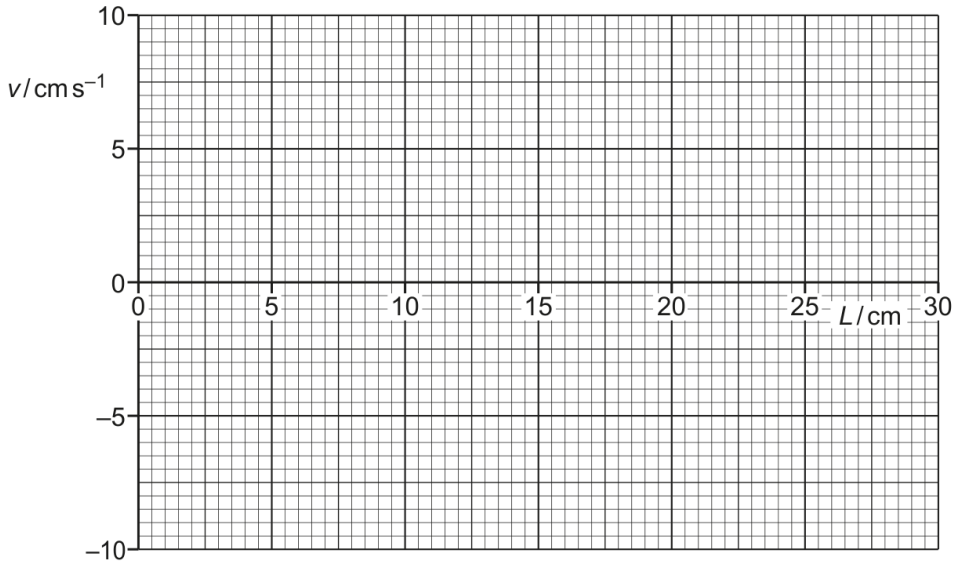


Fig. 4.3

[3]

15 A trolley on a smooth surface is attached by springs to fixed blocks as shown in Fig. 4.1.

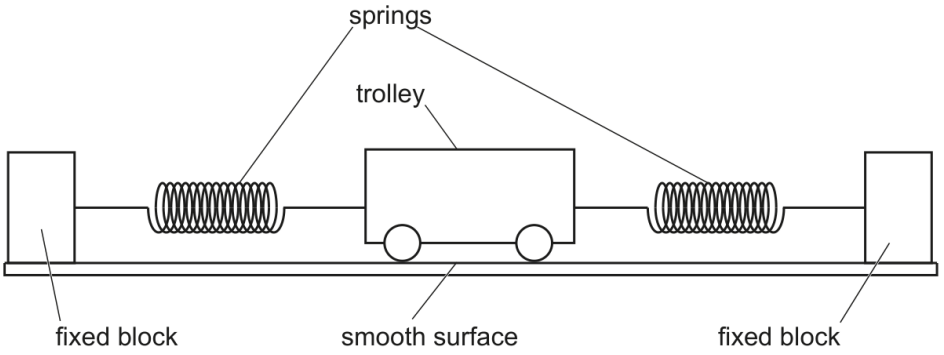


Fig. 4.1

The trolley oscillates horizontally about its equilibrium position with an amplitude of 12 cm. Fig. 4.2 shows the variation of the acceleration a of the trolley with displacement x from its equilibrium position. Friction between the trolley and the surface can be assumed to be negligible.

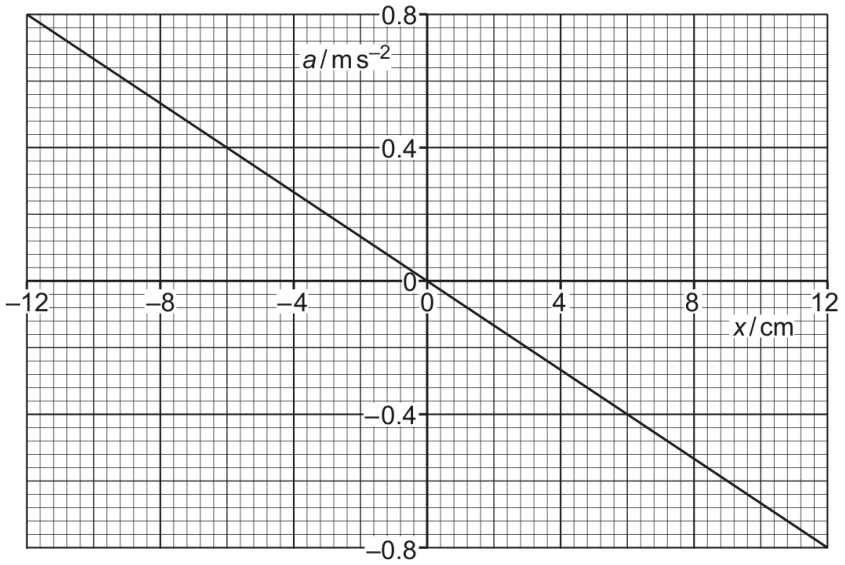


Fig. 4.2

(a) Describe the features of the line in Fig. 4.2 that demonstrate that the motion of the trolley is simple harmonic.

.....

.....

..... [2]

(b) Use Fig. 4.2 to determine the period T of the oscillations of the trolley.

T = ... s [3]

(c) **(i)** On the line of the graph of Fig. 4.2, label with the letter P one point where the kinetic energy of the trolley is zero. [1]

(ii) On the line of the graph of Fig. 4.2, label with the letter Q an approximate position of one point where the kinetic energy of the trolley is equal to the potential energy stored in the springs. [1]

16 The piston in the cylinder of a car engine moves in the cylinder with simple harmonic motion. The piston moves between a position of maximum height in the cylinder to a position of minimum height, as illustrated in Fig. 3.1.

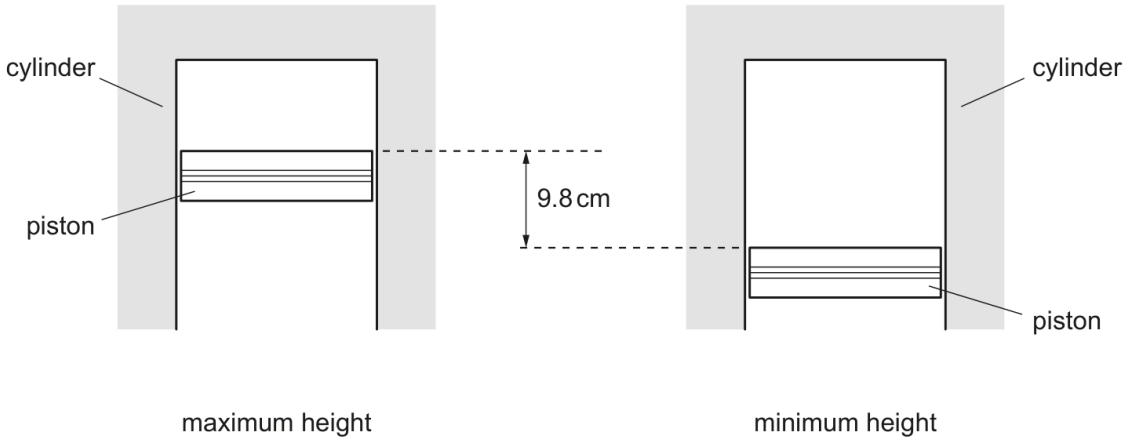


Fig. 3.1

The distance moved by the piston between the positions shown in Fig. 3.1 is 9.8 cm.

The mass of the piston is 640 g.

At one particular speed of the engine, the piston completes 2700 oscillations in 1.0 minute.

(a) For the oscillations of the piston in the cylinder, determine:

(i) the amplitude

amplitude = ... cm [1]

(ii) the frequency

frequency = ... Hz [1]

(iii) the maximum speed

maximum speed = ... $m\,s^{-1}$ [2]

(iv) the speed when the top of the piston is 2.3 cm below its maximum height.

speed = ... $m\,s^{-1}$ [2]

(b) The acceleration of the piston varies.

Determine the resultant force on the piston that gives rise to its maximum acceleration.

force = ... N [3]

17 A dish is made from a section of a hollow glass sphere.

The dish, fixed to a horizontal table, contains a small solid ball of mass 45 g, as shown in Fig. 4.1.

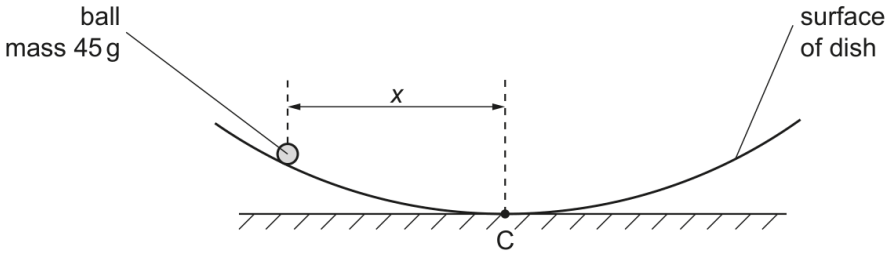


Fig. 4.1

The horizontal displacement of the ball from the centre C of the dish is x.

Initially, the ball is held at rest with distance x = 3.0 cm.

The ball is then released. The variation with time t of the horizontal displacement x of the ball from point C is shown in Fig. 4.2.

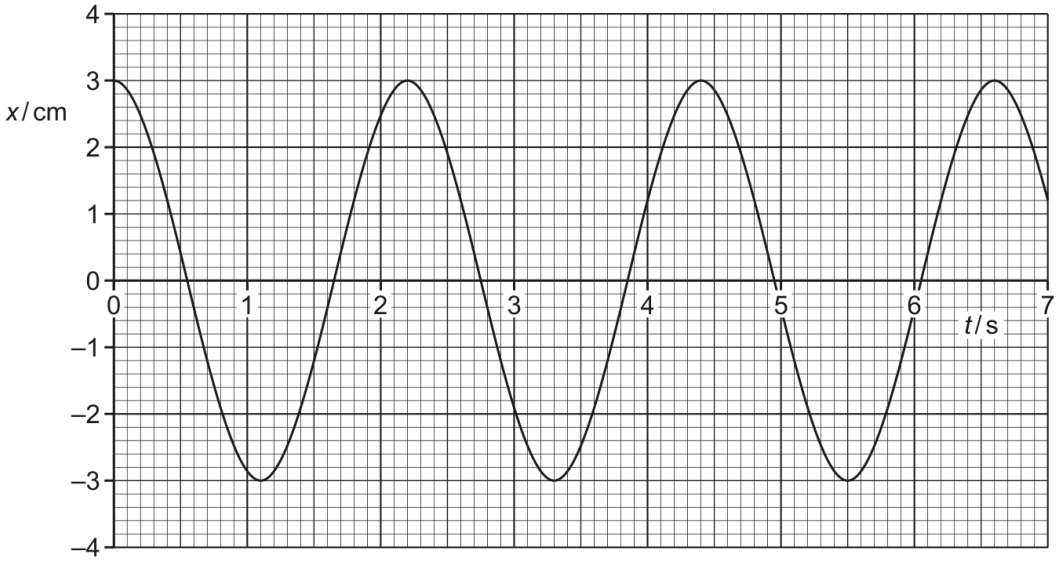


Fig. 4.2

The motion of the ball in the dish is simple harmonic with its acceleration a given by the expression

$$a = -\left(\frac{g}{R}\right)x$$

where g is the acceleration of free fall and R is a constant that depends on the dimensions of the dish and the ball.

(a) Use Fig. 4.2 to show that the angular frequency ω of oscillation of the ball in the dish is $2.9\,\text{rad}\,\text{s}^{-1}$.

[1]

(b) Use the information in **(a)** to:

(i) determine R

R = ... m [2]

(ii) calculate the speed of the ball as it passes over the centre C of the dish.

speed = ... $\text{m}\,\text{s}^{-1}$ [2]

18 A pendulum consists of a metal sphere P suspended from a fixed point by means of a thread, as illustrated in Fig. 3.1.

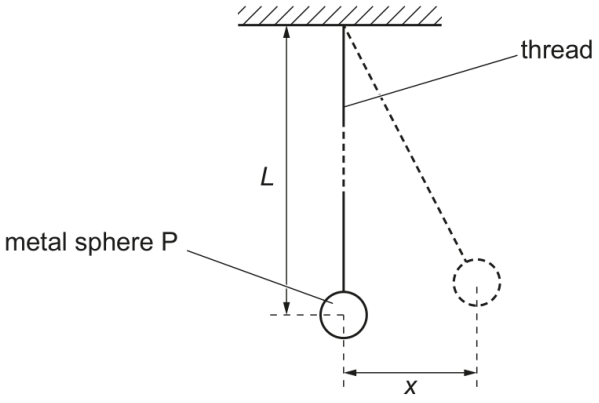


Fig. 3.1

The centre of gravity of sphere P is a distance *L* from the fixed point.

The sphere is pulled to one side and then released so that it oscillates. The sphere may be assumed to oscillate with simple harmonic motion.

(a) State what is meant by *simple harmonic motion*.

.....

.....

..... [2]

(b) The variation of the velocity v of sphere P with the displacement x from its mean position is shown in Fig. 3.2.

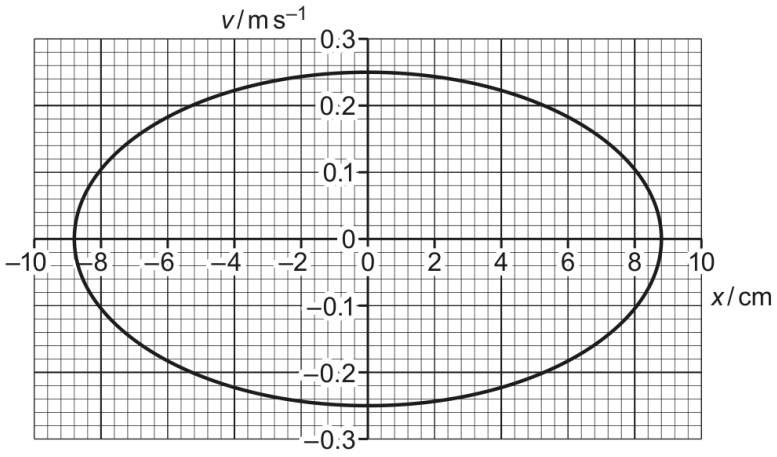


Fig. 3.2

Use Fig. 3.2 to determine the frequency f of the oscillations of sphere P.

f = ... Hz [3]

(c) The period T of the oscillations of sphere P is given by the expression

$$T = 2\pi\sqrt{\left(\frac{L}{g}\right)}$$

where g is the acceleration of free fall.

Use your answer in **(b)** to determine the length L.

L = ... m [2]

(d) Another pendulum consists of a sphere Q suspended by a thread. Spheres P and Q are identical. The thread attached to sphere Q is longer than the thread attached to sphere P.

Sphere Q is displaced and then released. The oscillations of sphere Q have the same amplitude as the oscillations of sphere P.

On Fig. 3.2, sketch the variation of the velocity v with displacement x for sphere Q. [2]

19 A simple pendulum consists of a metal sphere suspended from a fixed point by means of a thread, as illustrated in Fig. 3.1.

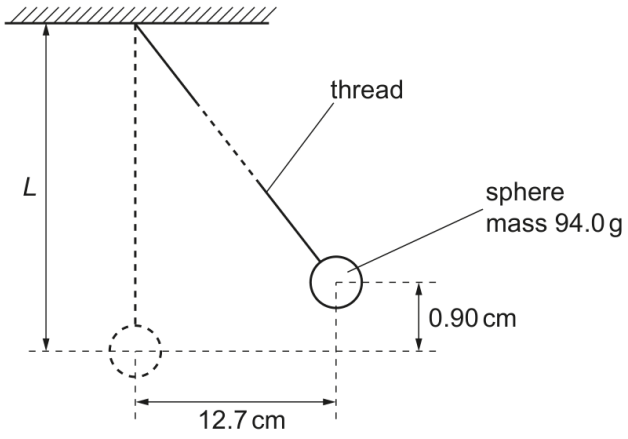


Fig. 3.1 (not to scale)

The sphere of mass 94.0 g is displaced to one side through a horizontal distance of 12.7 cm. The centre of gravity of the sphere rises vertically by 0.90 cm.

The sphere is released so that it oscillates. The sphere may be assumed to oscillate with simple harmonic motion.

(a) State what is meant by *simple harmonic motion*.

.....

.....

..... [2]

(b) **(i)** State the kinetic energy of the sphere when the sphere returns to the displaced position shown in Fig. 3.1.

(ii) Calculate the total energy E_T of the oscillations.

(iii) Use your answer in **(ii)** to show that the angular frequency ω of the oscillations of the pendulum is $3.3\,\text{rad}\,\text{s}^{-1}$.

(c) The period T of oscillation of the pendulum is given by the expression

$$T = 2\pi\sqrt{\left(\frac{L}{g}\right)}$$

where g is the acceleration of free fall and L is the length of the pendulum.

Use data from **(b)** to determine L.

L = ... m [3]

20 A hollow tube, sealed at one end, has a cross-sectional area A of 24 cm^2.
The tube contains sand so that the total mass M of the tube and sand is 0.23 kg.

The tube floats upright in a liquid of density ρ, as illustrated in Fig. 3.1.

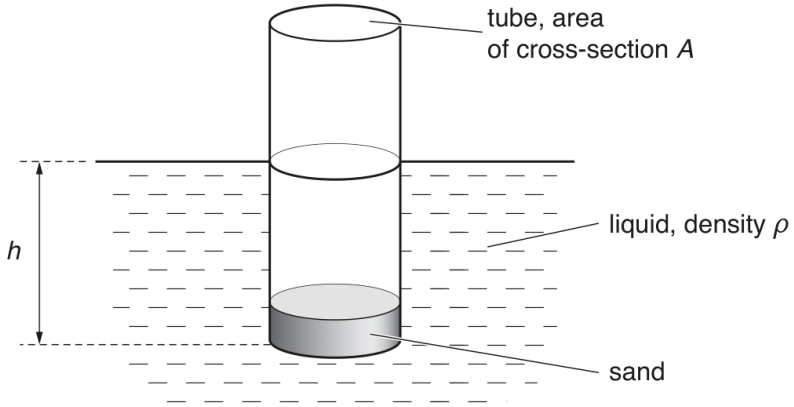


Fig. 3.1

The depth of the bottom of the tube below the liquid surface is h.

The tube is displaced vertically and then released. The variation with time t of the depth h is shown in Fig. 3.2.

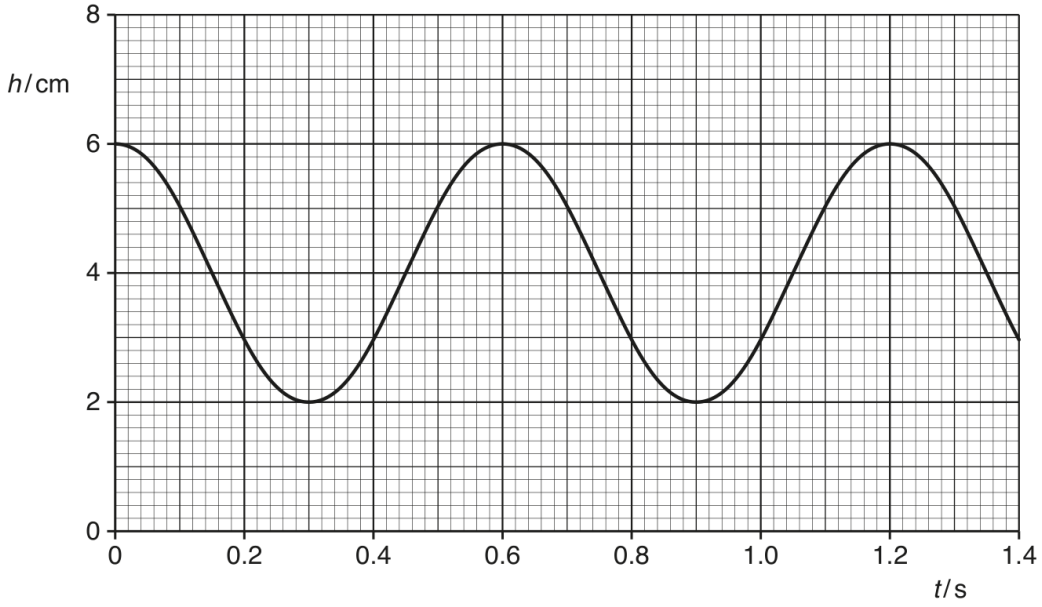


Fig. 3.2

(a) Determine:

(i) the amplitude, in metres, of the oscillations

amplitude = ... m [1]

(ii) the frequency of oscillation of the tube in the liquid

frequency = ...Hz [2]

(iii) the acceleration of the tube when h is a maximum.

acceleration = ...m s^{-2} [2]

(b) The frequency f of oscillation of the tube is given by the expression

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{A\rho g}{M}\right)}$$

where g is the acceleration of free fall.

Calculate the density ρ of the liquid in which the tube is floating.

ρ = ...kg m^{-3} [2]

21 A spring is hung vertically from a fixed point. A mass M is hung from the other end of the spring, as illustrated in Fig. 3.1.

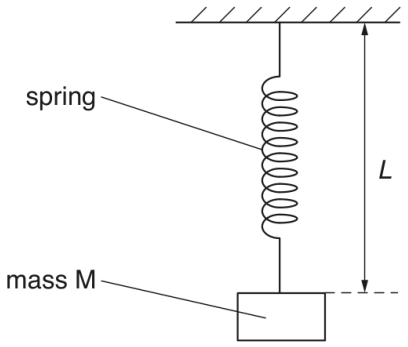


Fig. 3.1

The mass is displaced downwards and then released. The subsequent motion of the mass is simple harmonic.

The variation with time *t* of the length *L* of the spring is shown in Fig. 3.2.

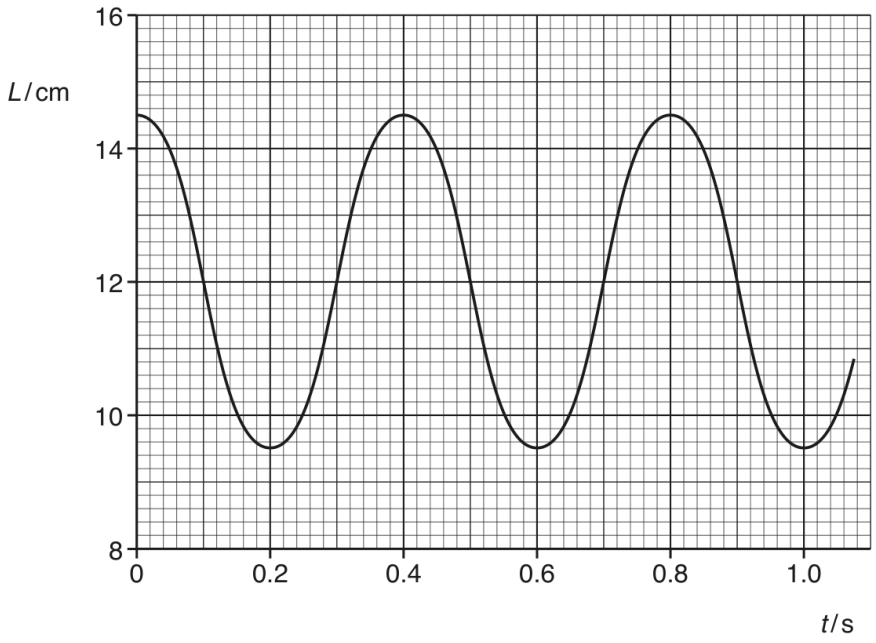


Fig. 3.2

(a) State:

(i) one time at which the mass is moving with maximum speed

time = ... s [1]

(ii) one time at which the spring has maximum elastic potential energy.

time = ... s [1]

(b) Use data from Fig. 3.2 to determine, for the motion of the mass:

(i) the angular frequency ω

ω = ... rad s^{-1} [2]

(ii) the maximum speed

maximum speed = ... m s^{-1} [2]

(iii) the magnitude of the maximum acceleration.

maximum acceleration = ... m s^{-2} [2]

(c) The mass M is now suspended from two springs, each identical to that in Fig. 3.1, as shown in Fig. 3.3.

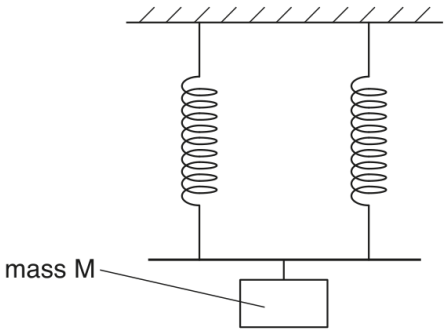


Fig. 3.3

Suggest and explain the change, if any, in the period of oscillation of the mass. A numerical answer is not required.

.....

.....

.....[2]

22 A mass is suspended vertically from a fixed point by means of a spring, as illustrated in Fig. 4.1.

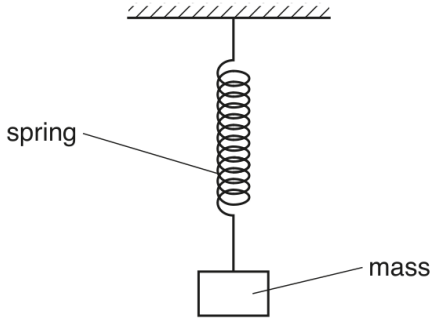


Fig. 4.1

The mass is oscillating vertically. The variation with displacement x of the acceleration a of the mass is shown in Fig. 4.2.

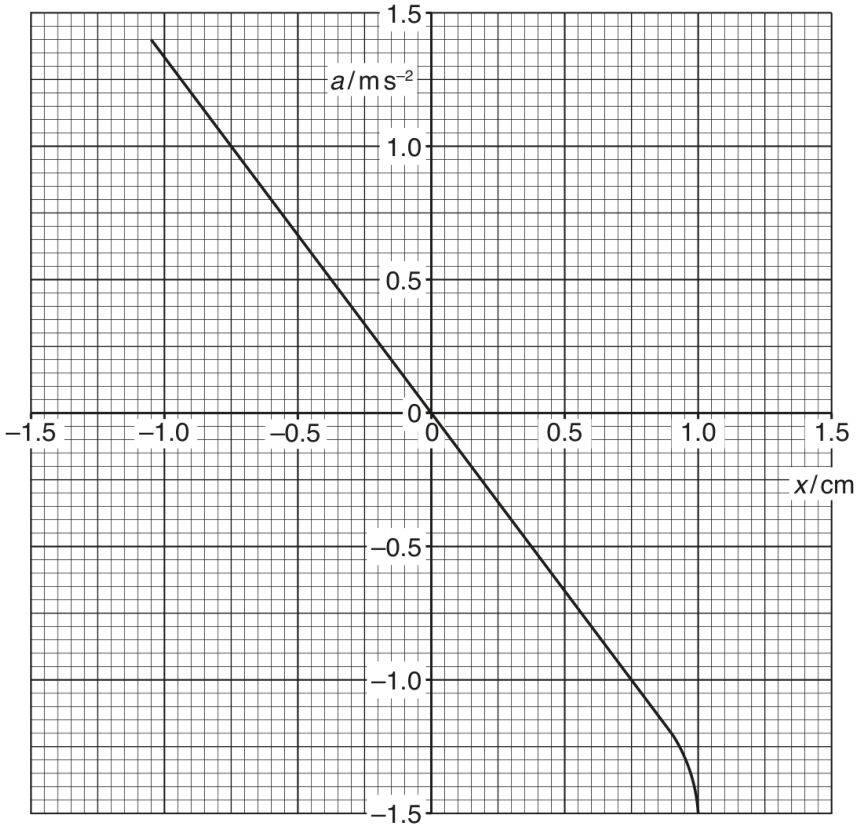


Fig. 4.2

(a) (i) State what is meant by the *displacement* of the mass on the spring.

.....

..... [1]

(ii) Suggest how Fig. 4.2 shows that the mass is not performing simple harmonic motion.

.....

..... [1]

(b) (i) The amplitude of oscillation of the mass may be changed.

State the maximum amplitude x_0 for which the oscillations are simple harmonic.

x_0 = ... cm [1]

(ii) For the simple harmonic oscillations of the mass, use Fig. 4.2 to determine the frequency of the oscillations.

frequency = ... Hz [3]

(c) The maximum speed of the mass when oscillating with simple harmonic motion of amplitude x_0 is v_0.

On Fig. 4.3, show the variation with displacement x of the velocity v of the mass for displacements from $+x_0$ to $-x_0$.

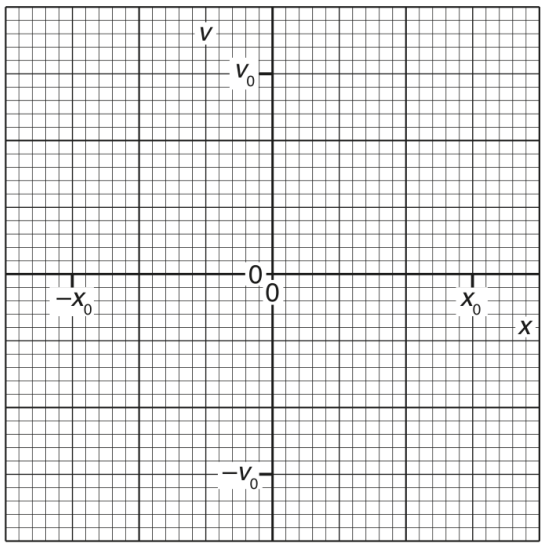


Fig. 4.3

[2]

23 A ball of mass *M* is held on a horizontal surface by two identical extended springs, as illustrated in Fig. 4.1.

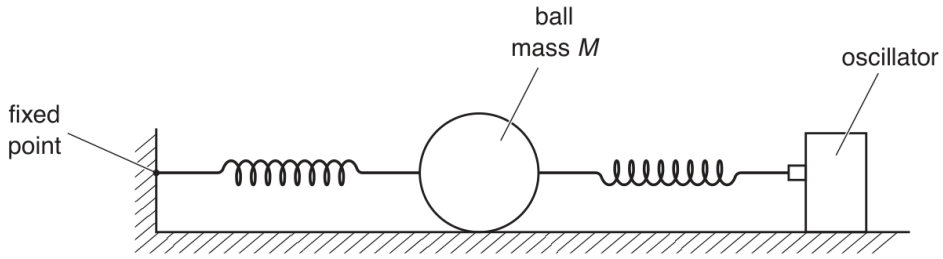


Fig. 4.1

One spring is attached to a fixed point. The other spring is attached to an oscillator.

The oscillator is switched off. The ball is displaced sideways along the axis of the springs and is then released. The variation with time *t* of the displacement *x* of the ball is shown in Fig. 4.2.

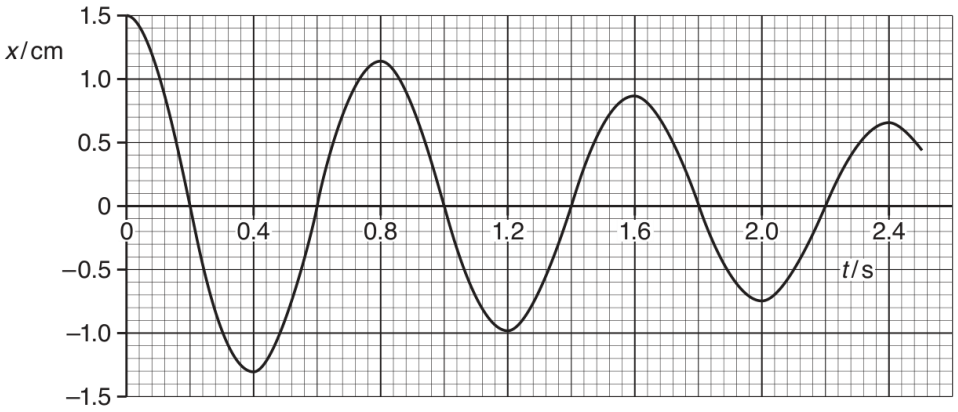


Fig. 4.2

(b) The acceleration *a* and the displacement *x* of the ball are related by the expression

$$a = -\left(\frac{2k}{M}\right)x$$

where *k* is the spring constant of one of the springs.

The mass *M* of the ball is 1.2 kg.

(i) Use data from Fig. 4.2 to determine the angular frequency ω of the oscillations of the ball.

ω = ... rad s^{-1} [2]

(ii) Use your answer in **(i)** to determine the value of k.

$k =$ ..... N m^{-1} [2]

24 A metal plate is made to vibrate vertically by means of an oscillator, as shown in Fig. 2.1.

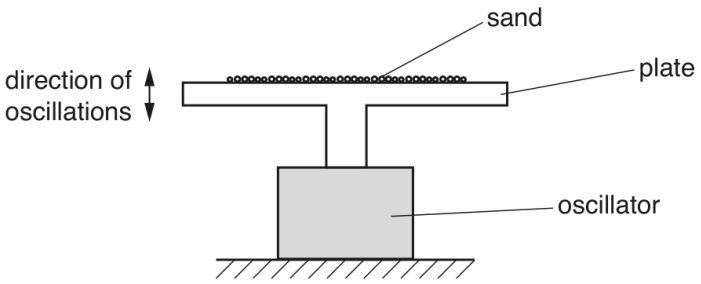


Fig. 2.1

Some sand is sprinkled on to the plate.

The variation with displacement y of the acceleration a of the sand on the plate is shown in Fig. 2.2.

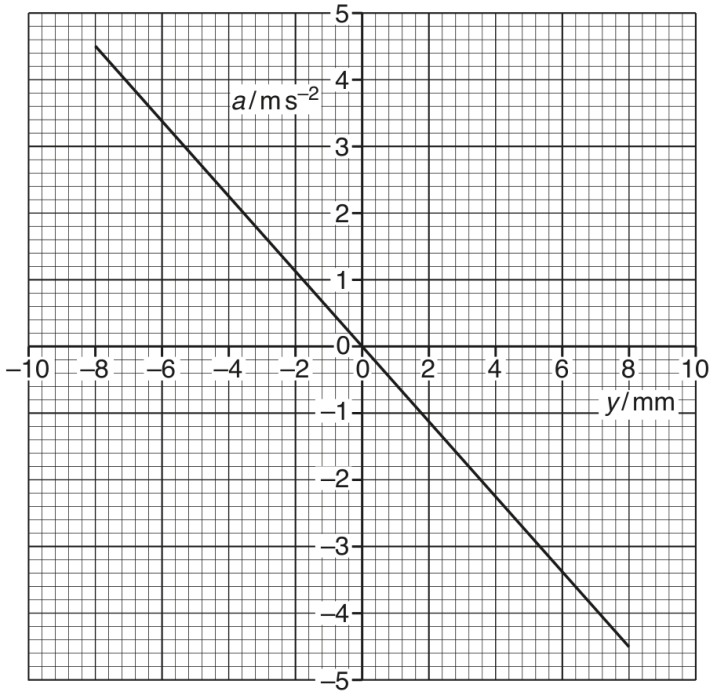


Fig. 2.2

(a) (i) Use Fig. 2.2 to show how it can be deduced that the sand is undergoing simple harmonic motion.

.....

.....

.....

.....[2]

(ii) Calculate the frequency of oscillation of the sand.

frequency = ... Hz [2]

(b) The amplitude of oscillation of the plate is gradually increased beyond 8 mm. The frequency is constant.

At one amplitude, the sand is seen to lose contact with the plate.

For the plate when the sand first loses contact with the plate,

(i) state the position of the plate,

.....[1]

(ii) calculate the amplitude of oscillation.

amplitude = ... mm [3]

25 **(a)** State two conditions necessary for a mass to be undergoing simple harmonic motion.

1. .....

.....

2. .....

.....

[2]

(b) A trolley of mass 950 g is held on a horizontal surface by means of two springs attached to fixed points P and Q, as shown in Fig. 4.1.

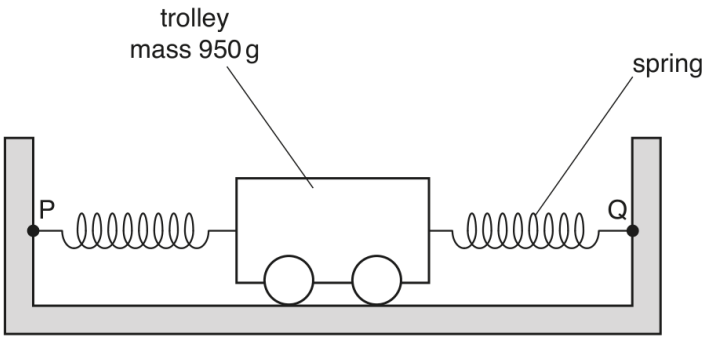


Fig. 4.1

The springs, each having a spring constant k of 230 N m^{-1}, are always extended.

The trolley is displaced along the line of the springs and then released.
The variation with time t of the displacement x of the trolley is shown in Fig. 4.2.

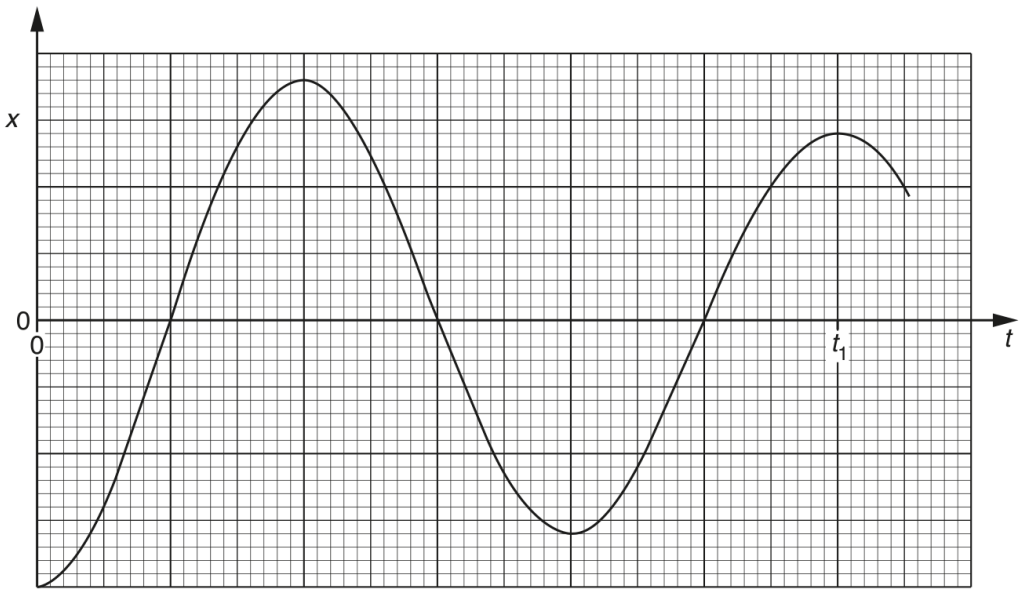


Fig. 4.2

(ii) The acceleration a of the trolley of mass m may be assumed to be given by the expression

$$a = -\left(\frac{2k}{m}\right)x.$$

1. Calculate the angular frequency ω of the oscillations of the trolley.

ω = ... rad s^{-1} [3]

2. Determine the time t_1 shown on Fig. 4.2.

t_1 = ... s [2]

26 A U-tube contains liquid, as shown in Fig. 3.1.

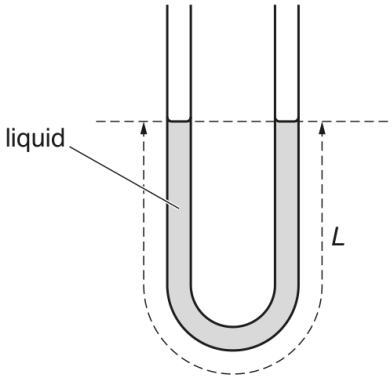


Fig. 3.1

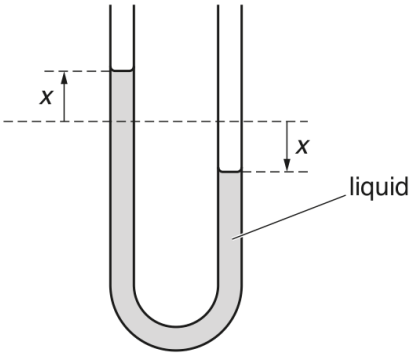


Fig. 3.2

The total length of the column of liquid in the tube is L.

The column of liquid is displaced so that the change in height of the liquid in each arm of the U-tube is x, as shown in Fig. 3.2.

The liquid in the U-tube then oscillates with simple harmonic motion such that the acceleration a of the column is given by the expression

$$a = -\left(\frac{2g}{L}\right)x$$

where g is the acceleration of free fall.

(a) Calculate the period T of oscillation of the liquid column for a column length L of 19.0 cm.

T = ... s [3]

27 A U-tube contains liquid, as shown in Fig. 4.1.

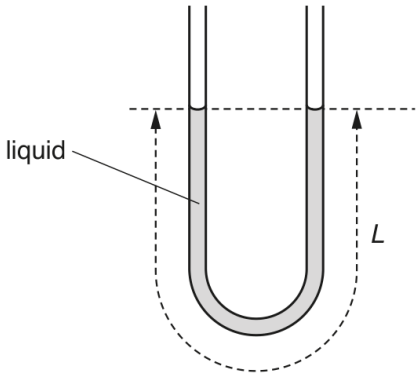


Fig. 4.1

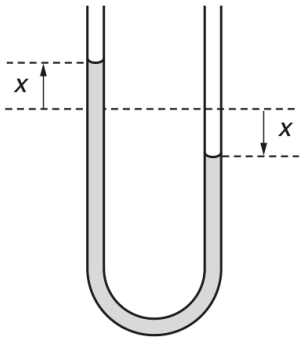


Fig. 4.2

The total length of the liquid column is *L*.

The column of liquid is displaced so that the change in height of the liquid level from the equilibrium position in each arm of the U-tube is *x*, as shown in Fig. 4.2.

The liquid in the U-tube then oscillates such that its acceleration *a* is given by the expression

$$a = -\left(\frac{2g}{L}\right)x$$

where *g* is the acceleration of free fall.

(a) Show that the liquid column undergoes simple harmonic motion.

[2]

(b) The variation with time t of the displacement x is shown in Fig. 4.3.

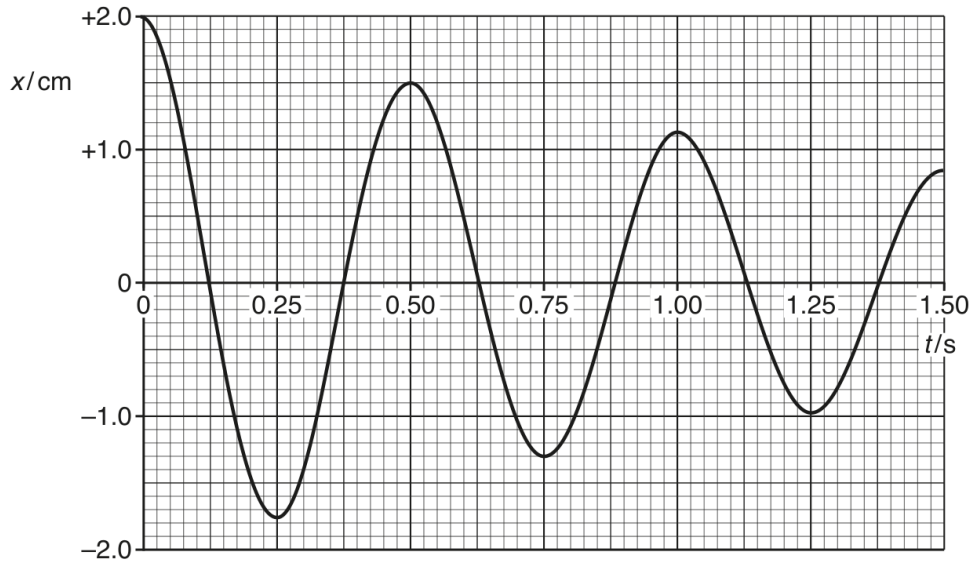


Fig. 4.3

Use data from Fig. 4.3 to determine the length L of the liquid column.

L = ... m [3]